

LECTURE 5: ECONOMIC GROWTH (AND CHEAP OIL)

Robert U. Ayres
INSEAD, Boulevard de Constance
F-77305 Fontainebleau Cedex
France
Email: robert.ayres@insead.edu

Introduction

In this lecture I suggest that the primary missing ingredient in growth theory (and for that matter in much of macro-economic theory) is the role of natural resources, materials, energy and work. It is curious, in my view, that most neoclassical growth models assume a unidirectional causality, viz. that natural resource consumption and use are strictly determined by the level of economic activity, but that resource consumption – and its consequences, including declining costs of extraction and processing – do not affect economic growth in return. The origins of physical production in the neo-classical paradigm remain unexplained, since the only driving variables in the models are accumulations of abstract labor and capital. The possibility of a ‘virtuous circle’ or positive feedback cycle involving the exploitation of natural resources has, up to now, been neglected by growth theorists, even though the existence of such a feedback cycle (*Figure 1.*) seems to be intuitively obvious.

The first major economist to criticize neo-classical economics on thermodynamic grounds was Professor Nicolas Georgescu-Roegen (hereafter G-R), who is most famous for his 1971 book *The Entropy Law and the Economic Process*. In the past I have disagreed with G-R’s insistence that scarce material elements – like energy – cannot be recycled indefinitely, a proposition that he mistakenly elevated to the status of a “Fourth Law” of thermodynamics.¹ Having said this, I must add that G-R did have something very important to say. To put it in the fewest possible words, his key point was that – in contrast to the standard neoclassical view – the economic system is a *materials processing* system that converts high quality (low entropy) raw materials into goods and services, while disposing of, and dissipating, large and growing quantities of high entropy materials and energy waste (i.e. waste heat). The economic system of industrial countries is driven mainly by solar exergy, much of which currently comes from solar exergy captured and accumulated hundreds of millions of years ago as fossil fuels.

G-R also understood, and emphasized repeatedly, that economic goods are of material origin. It follows that processing the materials requires available energy (i.e. *exergy*). But for his insistence that “matter matters” (i.e. the so-called Fourth Law), he would probably have accepted the physicist’s view that exergy is the ‘ultimate resource’ e.g. (Goeller, Weinberg 1976) Moreover, most ‘services’ however immaterial they appear to be at the user interface, ultimately depend on material devices, machines and infrastructure.

How then can exergy and the Second Law play a central role in economics? Let me resort to a quotation here:

“It looks like environmental economics is faced with a profound dilemma: on the one hand, thermodynamics is highly relevant to environmental economics so that thermodynamic concepts seem to have to be integrated somehow to redress the deficiencies of neoclassical economics. On the other hand all approaches toward such an integration were found to be incomplete and unsatisfactory. On the basis of the neoclassical paradigm, thermodynamic constraints are able to take only the first law of thermodynamics into consideration, whereas the implications of the entropy law cannot be given due regard. But the radical alternative of an energy theory of value was even more of a failure...”(Söllner 1997) p.194.

There can be several views as to where Söllner’s negative assessment leaves us. For me, the answer is to incorporate exergy, and second-law efficiency, explicitly into an endogenous alternative to the neoclassical theory of economic growth. Indeed, the normative implication of Georgescu-Roegen’s world-view, slightly re-stated, is that – thanks to Second Law

irreversibility – it is essential to utilize scarce resources *more and more efficiently* in the future. In other words, increasing efficiency is the key to combining economic growth with long-term sustainability.

It follows that, if the economy is a ‘materials processor’, as I argue (*Figure 2*), then exergy flux or exergy services ought to be one of the factors of growth. I think that G-R would have agreed with this approach.

History of growth theory

Most economic theory since Adam Smith has assumed the existence of a *static* equilibrium between supply and demand. It is this equilibrium that permits the beneficent functioning of Adam Smith’s ‘invisible hand’. The notion was successively refined by Ricardo, Say, Walras, Wicksell, Edgeworth, Pareto and others in the 19th and early 20th centuries.

In the 1870s Leon Walras formulated the postulate as a competitive equilibrium in a multi-product system with stable prices where all product markets (and labor markets) ‘clear’, meaning no shortages and no surpluses (Walras 1874). He also postulated a sort of auction process, never really defined, known as *tatônnement*, by means of which prices are determined in a public manner, without individual pair-wise bargaining, such that all actors have perfect information. Walras’ proposition (that such an equilibrium is possible) was widely accepted, though not proved until long after his death (Wald 1936; Arrow, Debreu 1954) Since then most economists have assumed that the real economy is always in, or very close to, a Walrasian equilibrium (e.g. (Solow 1970)).

Unfortunately the Walrasian model was (and is) static. It applies only to exchange transactions, and does not attempt to explain either production or growth. Growth was an obvious fact of economic life, of course. It was attributed in the 19th century to labor force (i.e. population) growth and capital accumulation. The latter was attributed to surplus (Marx) or savings. The most influential models of the 1930s and ‘40s were based on a formula attributed to Fel’dman (Fel’dman 1964, 1928) equating the rate of growth of the economy to the savings rate divided by the capital-output ratio, or (equivalently) the ratio of annual savings to capital stock. The formula was ‘rediscovered’ by Roy Harrod and Evsey Domar (Domar 1946; Harrod 1939) These models, which emphasized the role of central planning (a relic of academic Marxism) dominated early postwar thinking about development economics.² For instance, a well-known text on development economics half a century ago states, without qualification, that “...the central fact of development is rapid capital accumulation (including knowledge and skills with capital)” (Lewis 1955). Development is just an euphemism for economic growth.

For a single-product, single sector model, modern growth theory began even earlier with Frank Ramsey (Ramsey 1928) Ramsey assumed an economy producing a single all-purpose capital and consumption good produced by homogeneous labor and the all-purpose good itself. The all-purpose good is necessarily abstract, immaterial and therefore mass-less. There is no role in the Ramsay model, or its successors, for conservation of mass, consumption of energy (exergy) or indeed for natural resources – or wastes and losses – of any kind.

It is possible to generate a sort of growth process in a multi-sector general equilibrium model, in which — as in the Ramsay case — all products are produced from other products in the system (von Neumann 1945). In this case capital goods can be segregated from consumption goods. In the von Neumann model, as in Ramsay, the rate of economic growth can be determined by the allocation between investment and consumption. But all goods are still abstract, immaterial and not subject to physical conservation laws. There is no extraction

of raw materials, consumption of energy (exergy) or disposal of wastes.

In the closed multi-product, multi-sector static economic system described by Walras (Walras 1874), Cassel (Cassel 1932), and Koopmans (Koopmans 1951), *every product is produced from other products made within the system, plus capital and labor services*. Von Neumann made the system 'grow' equally in all directions (sectors) – rather like a balloon – by the simple trick of increasing the output of all sectors uniformly (von Neumann 1945). Abstract flows of money and services are presumably exempt from the physical law of conservation of mass-energy. But that law – the First Law of Thermodynamics – guarantees that waste residuals must be pervasive, just as the Second Law guarantees that all economic processes are dissipative and irreversible and can only be maintained by a continuous flow of free energy (or exergy) from outside the system. Yet, the neo-classical conceptualization until the 1970s implied that wastes and emissions – if they exist at all – are exceptional. In this version of the theory waste residuals do not affect growth or decrease the wealth or welfare of society as a whole, and can be disposed of at no cost. That view is no longer possible.³

Yet the origins of physical production in the neoclassical paradigm remain unexplained, since the only explanatory variables are abstract labor and abstract immaterial capital (*Figure 3*). The realism of the core assumption – that only capital accumulation drives growth – was sharply challenged in the early 1950s. Research based on reconstructions of historical time series of the supposed factors of production (labor and capital) drastically reduced the apparent role of capital accumulation as a driver of economic growth (Abramowitz 1952, 1956; Fabricant 1954). For example, Fabricant estimated that capital accumulation only accounted for 10 percent of per capita US economic growth since the middle of the 19th century.

Most economists are still using versions of a theory of growth developed for a single-sector model nearly half a century ago by Robert Solow, who was awarded a Nobel Prize for his accomplishment (Solow 1956, 1957); also Swan (Swan 1956). The theory was developed further by Meade, another Nobel laureate (Meade 1961). A key feature of the Solow-Swan model was the explicit introduction of aggregate production functions in which capital services are derived from an artifact called 'capital stock'.⁴

Problems of defining and measuring capital gave rise to a well-known debate between Paul Samuelson and Robert Solow et al at MIT and Joan Robinson and others at Cambridge in the UK (Robinson 1953-54, 1955) and later reviewed by Harcourt (Harcourt 1972). Critics of the production function approach pointed out that firms in the real world, lacking perfect information, cannot 'move' along an aggregate production function, either singly or as an aggregate, as the theory implies. It was also argued that capital cannot be measured independently of its rate of return, as determined in the national accounts. The debate was never really settled by argument. However the production function approach seems to have triumphed in the sense that it is widely used in practice.

The Solow model, in its simple form, depends only on two variables (Solow 1956, 1957) They are total labor inputs and total capital stocks. (Labor and capital services are assumed to be proportional to the corresponding stock). However, as the work of Fabricant and Abramowitz had already showed, the two explanatory independent variables – or factors of production – did not explain the observed growth of the US economy. The unexplained 'Solow residual' accounted for over 80 % of the per capita growth in output. Solow named this residual 'technological progress' and introduced it as an exogenous multiplier of the production function. The multiplier is usually expressed as an exponential function of time which increases at a constant average rate based on past history. The multiplier is now called "total factor productivity" (TFP).

Of course, naming a disease is not the same as explaining it. Nevertheless, thanks to

the miracle of differential calculus, it is standard to speak of the productivity of labor, the productivity of capital and (in some circles) the productivity of resources. Productivity estimation has become a mini-industry (Kendrick 1956, 1961, 1973; Gollop, Jorgenson 1980; Kendrick, Grossman 1980; Hogan, Jorgenson 1991). Some economists have made careers of decomposing observed productivity in terms of other variables e.g. (Denison 1962, 1967, 1974, 1985). This activity is called 'growth accounting'. However, growth accounting is not an explanatory theory of growth.

Drawbacks of the current neo-classical approaches

Apart from a number of other questionable simplifications, the standard Solow-Swan theory suffers from a crucial and recognized deficiency: it cannot explain the main driver of economic growth. Unfortunately there has never been any real theory to explain technical progress. Notwithstanding fancy packaging and the use of enormously sophisticated 'computable general equilibrium' (CGE) algorithms, virtually all economic projection models nowadays are still driven by single-sector Solow-type models using either Cobb-Douglas or CES production functions of capital and labor.⁵ These models always assume some underlying long-term rate of productivity increase, *while simultaneously remaining in Walrasian (hence static) equilibrium*. As I have pointed out above, growth not explainable by an accumulation of the two factors of production, namely reproducible capital stock, and human capital stock, is usually attributed to a stock of technological 'knowledge' that grows smoothly (and costlessly), of its own accord.

There are serious problems with the neoclassical growth-in-equilibrium assumption. It assumes that technical change is exogenous, uniform and smooth. In fact, it assumes that labor (and capital) become steadily and continuously more productive, while the economy remains all the time in equilibrium. However, it is evident that smooth, gradual change, uniform across all sectors – whether attributable to learning, experience or scale effects – *cannot* explain either technological or economic history. It is especially inconsistent with observed patterns of structural change that characterize the real world and would have to be reflected in multi-sector models.

Walrasian static equilibrium is clearly inconsistent with inventive activity or innovation at the micro-scale or structural change at the macro-scale. Thus growth-in-equilibrium is essentially impossible. Detailed critiques of the equilibrium assumption are hardly original with me, e.g. see (Kaldor 1971; Kornai 1973).

The standard growth model has other drawbacks. For instance Solow-Swan theory had a built-in tendency for declining productivity due to declining returns to capital investment. This is because the capital stock eventually becomes so large that annual investments (from savings) are comparatively insignificant or may be needed simply to compensate for annual depreciation. When this point of 'capital saturation' is reached, further growth per capita can only result from 'technical progress' or TFP, which (as noted above) is unexplained.

This feature of the Solow model implies that countries with a small capital stock will grow faster than countries with a large capital stock. Thus the model also predicts gradual 'convergence' between poor and rich countries. There is some evidence for this in East Asia, but not in Africa or Latin America. A consequence of the saturation effect predicted by the model was that richer countries should grow slower, and developing countries should grow faster and gradually catch up to the more industrialized countries. In fact, economic growth in the industrialized countries has not slowed down to the degree suggested by the theory, while developing countries (with some notable exceptions) have not been catching up (Barro, Sala-I-Martin 1995).

In response to this perceived difficulty, some theorists have suggested that capital and labor augmentation – in the sense of quality improvements – might enable the Solow-Swan model to account for the observed facts. For instance, education and training should (and does) make the labor force more productive, and knowledge presumably does not depreciate as does most kinds of physical capital. Similarly, capital goods have become more productive as more advanced technology is embodied in more recent machines, thus compensating for depreciation. Augmentation of labor and capital, in some degree, is undoubtedly an observable and quantifiable fact. Allowing for it, a number of cross sectional econometric studies were carried out in the '90's to test this idea. Indeed, some of them seemed, at first, to provide empirical support for the idea that exogenous technological progress (TFP) can be eliminated from the theory and that factor accumulation alone could, after all, explain the observed facts of economic development (Mankiw, Romer, Weil 1992; Mankiw 1995, 1997; Young 1995; Barro, Sala-I-Martin 1995).

However more recent research has discredited that conclusion. It has reinstated the original Solow view that factor accumulation is not the central feature of economic growth after all (Easterly, Levine 2001). Easterly and his colleagues, having extensively reviewed the published literature of economic development studies, argue – as Solow did – that “something else” accounts for most of the observable differences between growth experiences in different countries. They adopt the standard convention of referring to this “something else” as TFP. In this lecture I hope to cast some new light on the origins of this unexplained driver of growth.

As I have said, the theory as articulated by Solow and others does not allow for ‘real’ material flows in the production function. Production and consumption are abstractions, linked only by money flows, payments for labor, payments for products and services, savings and investment. These abstract flows are governed only by equilibrium-seeking market forces (the “invisible hand”). There is no deep fundamental connection in neo-classical theory between the physical world and the economy. The equilibrium assumption is needed mainly to justify the assumption that output is a function of capital and labor inputs and that the output elasticities of the factors of production (i.e. marginal productivities) should correspond to factor payment shares in the National Accounts.⁶ This ‘requirement’ is a consequence of the theory of income allocation between factors (capital and labor) in a population of perfectly competitive producers of a single all-purpose good, in equilibrium. This simplistic theory is described in every economics textbook, although it has almost no relevance to the real world. Luckily, in a three-factor multi-sector model with a sequential ‘chain’ structure, such as we propose, it can be shown that there is no correspondence between payment shares in the National Accounts and factor productivities (Ayres 2001).

The production function approach is generally coupled with an assumption of ‘constant returns to scale’ which essentially means that N copies of an economic system would produce exactly N times the output of one system. Putting it another way, a big country like the US not necessarily richer per capita than a small one like Switzerland or Sweden. This assumption is both defensible on the basis of empirical evidence and mathematically very convenient. In fact, it sharply limits the mathematical forms of allowable production functions to homogeneous functions of the first order, also known as the Euler condition. On the other hand, even if the strict constant returns to scale postulate is violated in the real world (i.e. if big economies grow slightly faster than small ones, *ceteris paribus*) the violation cannot be very great. In other words, while the factor productivities of a Cobb-Douglas production function might conceivably add up to slightly more than unity, the deviation cannot realistically be large.⁷

New 'endogenous' theories of growth

Solow's 1956-57 model (cited above) implies that capital should exhibit diminishing returns, i.e. that either savings and investment as a fraction of output must increase or the growth-rate must slow down as capital stock increases, since capital depreciation inevitably absorbs more and more of the available savings and investment. For the same reason it also implies that less developed economies will grow faster than more mature economies. As mentioned above, neither slowdown nor convergence has been observed as a general characteristic of the real world (Barro, Sala-I-Martin 1995). This fact (among others) stimulated interest in the late 1980s in new models capable of explaining continuous steady-state growth. They attempt to overcome the limitations of Solow's production function approach by modifying the traditional feature of diminishing returns to capital.

In response to this problem, neoclassical development economists began thinking about other possible ways to endogenize the standard theory without making drastic changes. Although not emphasized in neo-classical growth theory, there is an endogenous mechanism that can explain a part of this residual, i.e. beyond that which is accounted for by labor and capital accumulation. The part that can be explained *without* radical (structure changing) technological innovations is due to learning, scale, and the accumulation of knowledge that leads to cost savings and product improvements. As explained previously, the mechanism in question is a simple positive feedback between increasing consumption, investment, increasing scale and 'learning-by-doing' or 'learning by using' at the societal level. This feedback results in declining costs leading to declining prices. Lower prices for goods and services stimulate further increases in demand and investment to increase the supply capacity. Increasing capacity gives rise to further economies of scale, which drive costs down. Moreover, production experience itself yields efficiency gains due to learning.

However, the dominant neoclassical endogenous growth theories now in the literature do not explicitly depend upon feedback. On the contrary, they are all 'linear' in the sense that they assume a simple uni-directional causal mechanism. The endogenous theory literature can be subdivided into three branches. The first is the so-called AK approach, harking back to the older Harrod-Domar 'AK' formalism mentioned above. In the newer version capital K is taken to include human capital (hence population and labor force). The growth of human capital is not subject to declining returns – as in the Solow model – because of the supposed (exactly) compensating influence of factor augmentation and technology spillovers. Spillovers are, of course, externalities, which suggests that the economic system need not be in perpetual equilibrium. Of course, this undermines the use of computable general equilibrium (CGE) models.

Neo-AK models began with Paul Romer (Romer 1986, 1987, 1990). Romer postulated a tradeoff between current consumption and investment in 'knowledge', which he assumes could be monopolized long enough to be profitable to the discoverer, but yet almost immediately becomes available as a free good (spillover) accessible to others (Romer 1986). A closely related approach, by Robert Lucas (Lucas 1988), based on some ideas of Uzawa (Uzawa 1962) focuses instead on 'social learning' and the tradeoff between consumption and the development of 'human capital'. In the Lucas version the spillover is indirect: the more human capital the society possesses, the more productive its individual members will be. This externality is embedded in the production function itself, rather than in the knowledge variable.

Other contributors to this literature divide capital explicitly into two components two kinds of capital, 'real' and human (King, Rebelo 1990). An alternative version assumes one kind of capital but two sectors, one of which produces only capital from itself. Another

approach was to allow increasing returns by preserving the distinction between cumulable and non-cumulable factors (e.g. labor, land) and modifying the production function to prevent capital productivity from vanishing even with an infinite capital/labor ratio e.g. (Jones, Manuelli 1990).

The second approach to endogenous growth theory emphasizes active and deliberate knowledge creation. This is presumed to occur as a result of maximizing behavior (e.g. R&D). Knowledge is assumed to be inherently subject to spillovers and dependent on the extent to which benefits of innovation can be appropriated by rent-seeking Schumpeterian innovators. Most models assume that inventors and innovators have negligible success at appropriating the benefits of their efforts. In other words, spillovers are essentially immediate and automatic. This assumption appears to be realistic (Nordhaus 2001)).

The development of endogenous growth theory along neo-classical lines seems to have culminated, for the present, with the work of Aghion and Howitt (Aghion, Howitt 1992, 1998) and Barro and Sala-I-Martin (Barro, Sala-I-Martin 1995). The former have pioneered a 'neo-Schumpeterian approach' emphasizing the research-driven displacement of older sectors by newer ones. This is essentially equivalent to the process of *creative destruction* originally described by Schumpeter (Schumpeter 1912, 1934). These authors (like Romer) focus on investment in knowledge itself (education, R&D) as a core concept.

The neo-classical endogenous theory has interesting features, some of which are shared by the Ayres-Warr theory, discussed hereafter. However, all of the so-called endogenous growth models share a fundamental drawback: they are and are likely to remain essentially theoretical because none of the proposed choices of core variables (knowledge, human capital, etc.) is readily quantified, and the obvious proxies (like education expenditure, years of schooling, and R&D spending) do not explain growth.

Evolutionary theory

The evolutionary approach emerged as a distinct branch of economic theory in the 1980s, although it was inspired by Schumpeter's early work (Schumpeter 1912, 1934). In standard neoclassical economics, competition in an exchange market near equilibrium is mainly driven by inherent comparative advantage (attributable to attributes of a location, for instance) or bargaining skill. In Schumpeter's world, by contrast, competition is driven by competitive advantage resulting from innovation by 'first movers', taking advantage of returns to experience and scale, returns to adoption, imperfect information, and (in some cases) legal monopolies during the life of a patent.

Neoclassical economists like Alchian and Friedman argued that Schumpeterian competition is consistent with profit maximization, because only maximizers will be 'selected' (in the Darwinian sense) by the market (Alchian 1950; Friedman 1953). This might be true in a static environment. But even in the biological case, where the environment changes relatively slowly, the work of Moto Kimura (1967) has shown that some mutations can spread through a population by random drift, without possessing any selective advantage (Kimura 1979). His theory of so-called *selective neutrality* is now conventional wisdom in population genetics.

In other words, if the selection mechanism is fairly slow and not very efficient, it is not necessary to optimize in order to survive, at least for a great many generations or in an isolated niche. Meanwhile, the environment and the conditions for competitive advantage change relatively quickly. If this is so in population genetics, why not in economics? We all know of inefficient firms that survive in isolated locations or specialized niches, simply because there is no nearby competition. In any case, Sydney Winter argued as long ago as

1964 that variation and selection need not bring about either optimality or equilibrium, whence predictions made on the basis of these postulates need not hold in the real world (Winter 1964). In later work Winter, working with Richard Nelson, pointed out that the Darwinian ‘selection’ analogy is inappropriate for economics because of the lack of an inheritance mechanism to assure perpetuation of whatever strategic behavior is successful at a point in time (Nelson, Winter 1982b; Winter 1984).

The main difference between evolutionary economics, as it has developed so far, and the neoclassical mainstream has been characterized as follows: that neoclassical theory postulates ‘representative’ firms operating on the boundary of a well-defined region in factor space, whereas evolutionary biology — and evolutionary economics — lays great stress on the existence of diversity (Van den Bergh 2003). In fact, the mechanism that drives the economic system, in the evolutionary view, is a kind of conflict between diversity and selection. In biology, diversity of populations and species is assured by mutation combined with diversity of environments. In economics diversity is the result of diversity of talents and ideas among entrepreneurs, together with diversity of competitors, institutional constraints, cultures and other external circumstances.

The selection mechanism in biology is called ‘survival of the fittest’, although the details of what constitutes ‘fitness’ are still very unclear, even a century and a half after the voyage of *The Beagle*. In economics there is no special term for whatever quality or competitive strategy is effective. However, it is generally assumed that one of the explicit strategies for survival is product or process innovation. Innovation is modeled as a search and selection process. Selection, in evolutionary economics, is essentially equated to survival into the next period as a viable competitor in the market (Nelson, Winter 1982a). These authors have shown that a plausible growth process can be simulated by postulating a population of firms (not in equilibrium), displaying bounded rationality, and interacting with each other on the basis of probabilistic rules.

However, Nelson and Winter share with mainstream economists a widespread view that the specific features of technological change are essentially unpredictable, except in the statistical sense that investment in R&D can be expected to generate useful new ideas. The contemporary orthodox view is reasonably well summarized by Heertje¹

“Technical knowledge, being the product of a production process in which scarce resources are allocated, can be produced. We do not know exactly what will be produced, but we are certain that we will know more after an unknown period”
(Heertje 1983)

The Nelson-Winter model of technological progress is essentially consistent with the view quoted above. In brief, it assumes (for convenience) that the probability of a successful innovation is a function of R&D investment and is more-or-less independent of past history or other factors. If discovery, invention and innovation were really so random, technological progress would be much smoother than it actually is.

Growth in the neo-classical paradigm: The standard model

The neoclassical paradigm, in its simplest version, does not allow any role for ‘real’ material flows, except as consequences (but not causes) of economic activity. It considers the economy as a kind of closed system in which production and consumption are linked by flows of money (wages flowing to labor and expenditures flowing to production). The goods and services produced are measured only in monetary terms. Of course the simplest version is too simple

for serious analysis, so it is normally modified and extended to include an investment component that produces capital. A still more elaborate version of the basic model can incorporate extraction and waste flows, but still only as abstractions without physical properties.

Since Solow's contribution in the 1950s the standard growth model has been a production function with two independent variables, capital (K), labor (L) and an exogenous multiplier $A(t)$ depending only on time, that represents technological progress or total factor productivity (TFP). The need for this multiplier arises from the fact noted at the beginning of this lecture, namely that the GDP has grown faster than either K or L or any combination of the two that satisfies the requirement of constant returns to scale, or Euler condition, namely that the function be homogeneous of the first order.

The simplest function that satisfies this condition is known as the Cobb-Douglas function, after its popularizers (Cobb, Douglas 1928) (*Figure 4*). Ignoring the time dependent 'quality' multipliers H , G , F for the moment (by arbitrarily setting them equal to unity), the original C-D function is proportional to $K^a L^b$ where the variables K , L can be regarded as index numbers normalized to unity at the beginning of the period (1900).

The constant returns (Euler) condition implies that the sum of the exponents should be unity, i.e. $a + b = 1$. The marginal productivities of the two factors are a and b respectively, as can be verified by direct calculation of the logarithmic derivatives. Solow equated those productivities with factor shares in the national accounts e.g. (Solow 1956, 1957). However, adding a third factor (resource inputs R , or commercial energy E) with an assumed productivity $(1 - a - b)$ unveils a difficulty. The apparent share of payments for raw materials (rents to resource owners) is clearly very small, as a fraction of GDP, which implies – under this interpretation – that the productivity of resource inputs is correspondingly very small. *Figure 5* graphs the key variables, for the US, over the period 1900-1998. *Figure 6* shows that the C-D function with a third variable but without an exogenous multiplier $A(t)$ does not explain historical US growth.

To be sure, the third factor is not truly independent of the other two. In particular, capital and resource flows are strongly synergistic. If the identification with factor payment shares is abandoned and the parameters a and b are determined econometrically, as in more recent models (e.g. McKibben, Wilcoxon 1994; McKibben, Wilcoxon 1995; Bagnoli, McKibben, Wilcoxon 1996) the imputed productivity of resource inputs is always much greater than the factor-payments share. (We arrive at a similar result subsequently by a different route.) Incidentally, it can be shown without difficulty (as a property of homogeneous functions) that relaxing the identification of resource factor share with its small share of payments in the national accounts – allowing a higher output elasticity – does not change the essential result.

There are other several other well-known mathematical forms in use, including the so-called 'constant elasticity of substitution (CES) function (Arrow, Chenery, Minhas, Solow 1961), the trans-log function (Jorgenson, Lau 1984) and the linear-exponential (LINEX) function (Kuemmel, Strassl, Gossner, Eichhorn 1985). In fact, these different mathematical forms are usually selected for convenience and not because any one of them is dramatically superior to the others. As noted already, it turns out that – regardless of which mathematical form is used – most of the growth in GDP must be attributed to the exogenously determined progress (or TFP) multiplier $A(t)$.

As it happens one can introduce a third variable – the usual choice is E for energy (exergy) or R for resources – in one of those production functions, while retaining the constant returns condition, in hopes of explaining growth without the multiplier $A(t)$. This has been done by various investigators, of which the first might have been (Allen, et al 1976).

Allen used the C-D form where a third variable simply adds a third exponent, while retaining the condition of constant returns and the assumption that factor (marginal) productivity corresponds to factor share. Of course resource inputs R have a very small factor share (no matter how they are calculated) and R 's inclusion did not make any significant difference in terms of explaining US growth over the long period from 1900 through 1998. The unexplained 'residual' is still dominant.

Later, other authors tried including E as a third factor, while relaxing the other two assumptions e.g. (Hannon, Joyce 1981; Kuemmel et al. 1985). However good fits to the data could only be achieved over short time periods.

The standard model then fits $A(t)$ independently to the unexplained residual that is called technological progress (or TFP). We have done this, as shown in *Figure 7*. The 'best fit' for the technical progress function is $A(t) = \exp[0.031(t - 1900)]$. In other words, throughout the twentieth century exogenous technical progress has averaged 3.1 percent per annum, with relatively small deviations above and below the average.

A growth model with useful work U as a third factor

A key feature of our (Ayres-Warr) approach is to treat the economic system as a materials-processing system that is constantly being pushed out of equilibrium by geo-political, socio-economic and — above all — technological innovations. In this respect our world-view is inconsistent with the standard neoclassical view, which assumes that energy and other naturally resource inputs must contribute very little to production because of their negligible role in the national accounts.

But, as we have also noted, the standard model contradicts economic intuition. Indeed, economic history suggests that increasing natural resource (energy) flows are indeed a major factor of production. Moreover, we emphasize that declining costs — in relation to the rising wages of labor — have induced ever-increasing substitution of machines (consuming fossil fuels) for human labor. *Figure 8*. Considering the positive feedback cycle (*Figure 1*) this long-term substitution has evidently been a key driver of economic growth, especially since the industrial revolution.

The approach we describe hereafter retains the aggregate production function, albeit in a more general form than the usual Cobb-Douglas or CES functions. It is a 3-factor function, in which the third factor (denoted U) refers not to resource inputs R (or E) but to 'useful work' performed by the economy, defined below. Once the factor-payments share argument is abandoned it makes sense — for reasons discussed in the next few paragraphs — to abandon R as well. The product of resource (exergy) inputs R times conversion efficiency f is equal to work, U . There are two cases for each variable, one of which includes biomass (agricultural and forest products) plus non-fuel minerals while the other version is limited to commercial fuels and energy sources.⁸ This variable can be introduced into the C-D production function discussed above (or any of the others). In the C-D case the values of the elasticity parameters a and b are then determined indirectly by a constrained ordinary least squares (OLS) fit. (To minimize problems associated with co-linearity this is normally done with incremental changes of logarithms of the variables, rather than on the variables themselves.)

Our choice of useful work U as a factor of production, rather than resource inputs R (or E), requires some further justification. As mentioned previously, we explicitly treat the economy as a materials processing system that evolves over time. We conceptualize this system as a chain of linked processing stages, starting with resource extraction, conversion, production of finished goods and services, final consumption (and disposal of wastes). It is understood that there are also feedbacks — reverse flows — along the chain. For instance,

capital goods are manufactured products that literally feed back into the extraction and processing stages. (This is the fundamental idea of Wassily Leontief's Input-Output model (Leontief 1936)). However, for the present we do not attempt to model these reverse flows of exergy services (useful work) explicitly, inasmuch as they are quantitatively small.

Retaining the chain conceptualization, each stage has physical inputs and physical outputs that pass to the next stage. From the global perspective the chain of process stages described in the previous paragraphs can be expressed as a product of successive conversion efficiencies, viz.

$$\begin{aligned} GDP &= R \times \frac{I_1}{R} \times \frac{I_2}{I_1} \times \frac{I_3}{I_2} \times \dots \times \frac{GDP}{I_n} \\ &= R \times f_1 \times f_2 \times \dots \times g \end{aligned} \quad (1.1)$$

where f_1 is the conversion efficiency of the resource (exergy) inflow R into the first level intermediate product, I_1 , f_2 is the conversion efficiency of I_1 to the second level intermediate product, I_2 and so forth. The term g is just the ratio of output to the last intermediate product. Equation 1.1 is still an identity. It only becomes a model when we specify the intermediate products and transformations.

As a first approximation, it is convenient to assume a two stage system with a single intermediate product, which we call U as already discussed above. We argue that this intermediate product can conveniently be identified as exergy services, or 'useful work'. Then

$$Y = Rfg = Ug \quad (1.2)$$

where f is the overall technical efficiency of conversion of 'raw' exergy inputs R to useful work output U .

While discarding the neoclassical equilibrium and optimality assumptions (as unnecessary), we retain the assumption that a production function of three factors (variables) is definable.⁹ We also retain the assumption of constant returns to scale, meaning that the production function must be a homogeneous function of the first order (Euler condition). Hence, the term g on the right-hand side of equation (1.2) can be interpreted as an aggregate production function provided it is a suitable homogeneous function of order zero, whose arguments are labor L , capital K , and useful work U .

The calculation of R and U for the US, in exergy terms, and the calculation of the efficiency factor f was a major undertaking in itself, since most of the underlying data are not published in official government statistics, but must be constructed laboriously from other time series and information about the history of technology. Details of these calculations cannot be presented here. However, the calculations of R and the efficiency of 'primary work' f (not including the 'secondary work' done by electrical devices) are described in a paper published in 2003 in the journal *Energy* (Ayres, Warr 2003). A further analysis of the efficiency of secondary (electrical) work appears in a more recent publication in the same journal (Ayres, Ayres, Pokrovsky 2005). Results are plotted in *Figures 9-10*.

The test of a theory is whether it can explain the past. Only then can one have confidence in its ability to predict the future. For a theory of growth, if one does not want to wait twenty or thirty years for confirmation, the best hope is to explain past economic growth for a very long period, such as a century. To do so we also need to specify a production

function, with as few independent parameters as possible, that fits (i.e. 'explains') historical data.

Our approach hereafter (inspired by Kuemmel) is to start by choosing plausible mathematical expressions for the output elasticities (marginal productivities) themselves (Kuemmel et al. 1985). To satisfy the Euler condition, these must be homogeneous functions of the independent variables. Since the elasticities are partial logarithmic derivatives of the output, by definition, one can perform the appropriate partial integrations to obtain the corresponding production function. There is just one small but critical difference between our scheme and that of Kuemmel *et al*, namely the substitution of useful work U (in our case) for commercial energy inputs E in their production function. The assumed marginal productivities are as follows:

$$\begin{aligned}\alpha &= a \left(\frac{L + U}{K} \right) \\ \beta &= a \left[b \left(\frac{L}{U} \right) - \left(\frac{L}{K} \right) \right] \\ \gamma &= 1 - \alpha - \beta\end{aligned}\quad (1.3)$$

The third term reflects the standard constant returns to scale (Euler) condition. Partial integration and exponentiation yields the following linear-exponential (LINEX) function:

$$Y = U \exp \left[a \left(2 - \left(\frac{L + U}{K} \right) \right) + ab \left(\frac{L}{U} - 1 \right) \right] \quad (1.4)$$

Comparing (1.4) with (1.2), it is clear that the function g becomes

$$g = \exp \left[a \left(2 - \left(\frac{L + U}{K} \right) \right) + ab \left(\frac{L}{U} - 1 \right) \right] \quad (1.5)$$

which is a zero-th order homogeneous function of the variables, as required for constant returns to scale.

With (1.4) as our production function, the constrained fitting procedure is actually simpler than in the Cobb-Douglas (C-D) case, where annual increments of logarithms of the variables are fitted to avoid auto-correlation. For the LINEX model we can treat U , L/U and $(L + U)/K$ as the three independent variables. (The variables L , K are taken from standard government publications. The variable U is not available from standard sources, but (as noted above) estimates of primary work for the US have been calculated (Ayres, Ayres, Warr 2003), while the secondary efficiency of electrical work has also been published (Ayres et al. 2005).

Taking logarithms of both sides of equation 1.4, it is easy to show that the error terms are stationary (trend-free), so it is possible to perform the constrained OLS fits more directly. It turns out that the combination of capital, labor and useful work as variables explains economic growth from 1900 to 2000 even better than the C-D production function. The best fit for the LINEX model, using the most inclusive definition of exergy input (including agricultural biomass) and useful output work (including animal work) is shown in *Figure 11*,

with and without the correction for electrical work.

Table 1: LINEX Parameters and Quality of Fit

Electrical Use Efficiency	a	b	RMSE
EXCLUDED	0.12	3.8	0.51
INCLUDED	0.09	4.5	0.49

The case where electrical work has been excluded was also in a recently published journal article (Ayres, Warr 2005).

Note that there is no longer any need for a time-dependent multiplier $A(t)$, whence the new model can be regarded as endogenous, albeit there are minor differences between actual and predicted GDP, especially after 1975 (ibid, fig. 8). We interpret this residual as (mostly) the marginal growth contribution from information and communications technology (ICT). However the details would take me beyond the scope of this lecture.) We have also tested the model recently on Japanese data, assuming US efficiencies, with equally good results, as shown in *Figure 12*.

Based on the simple theory and the empirical results exhibited above, it seems clear that *useful work done* (or *exergy services delivered*) can be regarded as a quality-adjusted factor of production, at least in the same sense that labor or capital can be so regarded. In effect, TFP or 'technical progress' can now be interpreted rather well in terms of the technical (thermodynamic) efficiency with which raw materials are converted into exergy services.

The corresponding (non-constant) marginal productivities for the US and Japan are plotted in *Figures 13-15*. *Figure 13* is also taken from (Ayres, Warr 2005)(fig. 9). Evidently when useful work is included as a third factor of production the corresponding productivity dominates, while the productivity of 'pure' labor exhibits a long-run decline to nearly zero. (In the Cobb-Douglas case, using constrained OLS, the fitted productivity of labor actually vanishes, yielding (in effect) a two-factor (capital-work) model.) To the extent that such a simple model can represent reality, the obvious interpretation would be that the gain in aggregate economic output resulting from an additional increment of unskilled labor, *ceteris paribus*, is now very small because there is now very little that an unskilled worker can do without either tools and machines (capital) or raw materials (exergy).

The implications for future economic growth

In order to forecast the future using any production function model it is necessary to forecast the factors. In the C-D case this means projecting capital investment (as a fraction of GDP) and the size of the labor force, based on independent demographic studies. While population and labor force ultimately bears some relationship to economic growth (whence the ideal model would be recursive in this variable), such effects are likely to be very long term and hence can be neglected to a first approximation. Exactly the same procedure applies in the LINEX case.

The only difference between the two cases is that in the C-D case (and in other neoclassical models) it is normal to project the technical progress function either from *Figure 7* (or from a similar function based on a shorter, more recent historical period) into the future. In effect this involves assuming that TFP continues to grow at the same average rate – or some lower rate, based on the forecaster's judgment – indefinitely.

In our case, by contrast, we need to forecast the product $U = fR$, where U is useful work, R is the resource (exergy) input and f is the efficiency with which it is converted into useful work. As already noted, the calculations, in practice, are not simple, because historical data on these variables is not easy to find. Actually, it is convenient to work with the ratio of useful work to GDP, or U/Y , which is plotted historically in *Figure 16*, taken from (Ayres, Warr 2005) fig.6. There are now two ways to proceed. The simplest is to make a straightforward linear extrapolation of the work curve in *Figure 16* and insert it directly into the LINEX function, along with extrapolations of capital and labor. Or, for a slightly more sophisticated approach, we can decompose U into its components, resource (exergy) inputs R (or E) and efficiency f and project them separately. Finally, one can make the model recursive by projecting 'exergy intensity' (R/Y) and utilizing a functional relationship of the form $Y(t) = F[Y(t-1)]$. The latter approach was used to derive the forecasts shown in *Figure 17*. This graph has been taken from a forthcoming paper (Warr, Ayres 2006).

The crucial difference between our forecast and the standard C-D forecasts is obvious at first glance. Our result suggests at least the distinct possibility that US economic growth may peak, depending on the rate of increase of the efficiency f in future decades. The standard model, of course, implies that economic growth will continue indefinitely, regardless of policies or technological progress.

On the forthcoming peak in global petroleum output

So far I have not considered the possibility of near-term resource depletion. However, such a possibility now exists with regard to liquid and gaseous hydrocarbon resources, especially petroleum. There is an interesting background story. In 1956, based on prior work on mineral ore depletion in Europe by D. F. Hewett (1929), geologist M. K. Hubbert predicted that US petroleum production would peak between 10 and 15 years after the date of the prediction [Hubbert 1956]. This prediction was made on the basis of a variety of data, including the fact that the rate of discovery of oil in the US (not including Alaska) had peaked in 1930 and declined sharply after 1940. In 1962 he refined his predictions, using additional data on discovery rates, reserves and production. He found that the rate of crude oil discoveries (including Alaska) had already peaked (in 1957), and that proved reserves were then at their peak (1962). Using a quantified version of Hewett's scheme, he predicted that peak production in the U.S. would occur in 1969. This proved to be accurate (the actual peak was 1970-71.) The Hubbert predictions were so disturbing to the oil industry that his methodology was very thoroughly criticized, in hopes of finding a flaw (e.g. [Menard 1981]). However, no serious flaw in the logic was found, or ever has been.

It seems that the CIA took Hubbert's methodology seriously and applied it to the USSR (Anonymous 1977). This report predicted that Soviet oil production would peak in the early 1980's. In fact there were two peaks, the first in 1983, at 12.5 million barrels per day and the second in 1988 at 12.6 barrels per day. Since then production has declined steadily. It seems likely that the Reagan administration, which took office in 1981, bearing in mind the economic havoc produced when US production peaked in 1981, followed by the Arab oil embargo and the "oil crisis" of 1973-74 and the deep recession that followed, decided to use the 'oil weapon' to destabilize the USSR. Reagan embarked on a major military buildup, putting the Soviet Union under pressure to keep up. Meanwhile, declining prices after 1981 forced the USSR to pump more oil to supply its clients in Eastern Europe and to sell in world markets for hard currency. Then in 1985 Reagan persuaded Saudi Arabia to flood the world markets with cheap oil. Again, the USSR had to increase output to earn hard currency. This led to the second peak in 1988. Two years later the USSR imploded (Heinberg 2004) pp 40-

41.

Application of the Hubbert methodology to the global scene is relatively straightforward, except that the data are considerably less reliable. However, the date of peak production is a function of the quantity of recoverable oil, which can remain as a variable.¹⁰ Hubbert himself estimated, on the basis of data available in the late 1960s, that the upper limit of the quantity of recoverable oil in the world (including oil already produced) was between 1350 and 2100 billion barrels of oil (BBO). In that case global peak output would have occurred shortly before the year 2000 [Hubbert 1969]. As it happens, his upper limit was a bit too pessimistic. A later study by the US Geological Survey (USGS), using another methodology (based on the distribution of known deposits in terms of specific gravity) concluded that cumulative world production to 1983 was 445 BBO, with demonstrated reserves of 723 BBO and a 90 percent probability that undiscovered reserves would lie between 321 and 1417 BBO, with a 'most likely' value of 550 BBO [Masters et al 1983]. That would correspond to a total ultimately recoverable quantity of 1718 BBO, well within Hubbert's earlier range.

As of late 1997 demonstrated world oil reserves were estimated at 1000 BBO, with another 550 BBO expected to be discovered (most likely value). Total cumulative global consumption had already reached 800 BBO, for an ultimate total of 2350 BBO [Hatfield 1997a]. On this basis, the halfway point could be reached in the first decade of the present century, or early in the second decade, depending on political and economic factors. The most recent USGS assessment of world undiscovered oil actually raises the previous estimates by 20% [USGS 2000], citing two new promising regions for exploration, the Greenland shelf and offshore Surinam. (No oil has yet been discovered, still less produced from either region, however).

However, there are also plausible reasons to believe that the above is over-optimistic. First among them is that in the two years 1988-89 four Persian Gulf countries and Venezuela increased their demonstrated reserve estimates by a total of 277 BBO, without reporting any discoveries of new oil fields! This could perhaps be justified on the basis of revised estimates of recoverability, although in several cases the upward revisions amounted to doubling (or more) of the original reserve figures. But the chances are good that this reported increase was largely jockeying for influence within OPEC and for purposes of discouraging investment in alternative sources of energy [Campbell 1991](Campbell 1997).

In any case, any upward revisions of recoverability of known fields should be re-allocated to the original discovery dates, not treated as if they were new discoveries. That this was not done strongly suggests ideological or political motivation. Another reason for skepticism is that – despite Yeltsin's 'opening of the Russian oil industry to foreign participation – production in the USSR, then the world's second largest producer, peaked in 1988. It has declined since then. Several other countries, including Iran, Iraq, Nigeria, Venezuela, Canada, Mexico and Indonesia have already passed peak production. Even the last super-giant field discovered, the North Sea has passed its apogee: the UK peaked in 1999, and Norway peaked in 2001. In consequence, Saudi Arabia's share of total world output is necessarily increasing because no other producer, even in the Middle East, has the capability of increasing its oil output rate significantly. Some doubt that the Saudis can do so either.

A third reason for believing that potential oil discoveries may have been overestimated for political or other reasons is that the global oil discovery rate has declined since the early 1960s, despite periods of high prices and intensive exploration. The optimism of a few years ago about the possibility of very large new 'supergiant' deposits under the Caspian Sea, or in Kazakhstan, has faded. New discoveries have not been spectacular. In fact, global

consumption has exceeded new discoveries since 1980 (Heinberg 2004) p.43. More to the point, oil prices declined in the mid-1980s and early 90s because high prices at the beginning of the period stimulated investment in energy conservation (especially in Japan) and simultaneously stimulated exploration.

Production capacity increased throughout the 1980s to around 125% of consumption by 1990 [Hatfield 1997b]. Actually the reserve pumping capacity hit 25% of global demand in 1985(Heinberg 2004). So prices declined drastically, to a low of \$10 per barrel. But by 1990 the global reserve in pumping capacity had fallen to 8% and by 1991 it was down to 2% (ibid). Then came the Iraqi attack on Kuwait, with the end result that Iraq's 3 million bbl/day was subtracted from output, but slow growth in Japan and the US kept demand from exploding. Then the US recovery after 1992 and rapid economic growth in Asia, especially China, have driven demand up sharply. China was an exporter in the '90s. It is now an importer. Prices rose until the economic crisis in Southeast Asia (1996-97), cut back demand again. The latest (2004-05) price rise was primarily the result of rapid economic recovery and growth in Asia, combined with continued growth in the US and cyclic recovery in Europe. Details apart, the crunch was predicted [ibid].

The 'age of oil' is not yet ended, to be sure. Optimists will be cheered by the latest USGS assessment of undiscovered world oil reserves, mentioned above, but several independent lines of argument suggest nevertheless that global peak production will occur near or soon after 2010 [Campbell & Laherrere 1997]. Prices, on the other hand, are likely to continue to be unstable, but the long term trend will be up rather than down. Of course rising prices would eventually bring new sources of 'non-conventional' oil into production, such as Greenland Shelf oil, Venezuelan heavy oil, Athabaska tar sands and Green River oil shale. The latter sources are potentially many times larger than the global stock of liquid petroleum. On the other hand they will be very much more expensive to extract and refine, both because of higher extraction costs and because refining of heavy oils required hydrogenation, and hydrogen has to be produced from hydrocarbons, especially natural gas.. Extremely large amounts of capital will be required. This creates a potential supply bottleneck, insofar as it may take a number of decades before new sources could reach significant output levels.(Needless to say, the efficiency of recovery of usable liquid fuels will be significantly lower and the exergy required for extraction and cracking will be much higher.) Even the most optimistic projections from the oil industry see non-conventional oil from all sources providing barely 10 mm barrels per day by 2030, and pessimists expect much less.

Natural gas is the fashionable alternative to liquid petroleum for the middle of the twenty- first century. However, it is interesting to note that the estimates of producible domestic gas that were made by energy companies and the US Geological Survey in the 1960s and 70s have undergone substantial reduction. Gas reserves have declined since the 1970s. In 1972 the USGS estimated exploitable gas resources in the US at 2100 trillion cubic feet (tcf). By 1989 this estimate had fallen eightfold to 263 tcf. Meanwhile reserves fell from 293 tcf in 1968 to 169 tcf 1992, despite intensive domestic exploration. The amount of domestic gas discovered per million feet of exploratory wells has declined for 50 years, despite improved technologies for identifying good places to drill. Production per well has declined from 500,000 cubic feet per day in 1972 to 185,000 cubic feet per day in 1992. As of the end of 2002 US gas reserves were either 183 tcf or 189 tcf, depending on the source (Darley 2004) p.211. Prices have risen sharply from less than \$2 per mcf in 1999 to over \$8 per mcf in 2002.

As with oil, the situation in the rest of the world lags several decades behind that in the US. For many years gas was a by-product of petroleum. As recently as 1986 about 5% of global gas produced (38% for the Middle East) was still vented or flared [Barns & Edmonds 1990, Figures 3.9 and 3.13]. This was due to lack of local markets or pipelines. However, the

unrecovered proportion of the gas has been declining, partly due to increased capacity to transport gas in liquid form and partly due to increased consumption for petrochemical manufacturing (of methanol, MBTE and ammonia) in Saudi Arabia. Projections made in 1990 suggest a current venting/flaring fraction at 3%. Gas wells are now being drilled for gas *per se*, independently of oil exploration. However, the geology is sufficiently similar to that for oil that the largest terrestrial and offshore gas fields have probably been discovered already. Global proven reserves as of 1991 amounted to 4000 trillion cubic feet, while mean estimates of undiscovered gas reserves were 5000 trillion cubic feet. Assuming normal growth in the gas market plus additional growth due to substitution for oil in some applications, this amounts to a century supply at zero growth rate, but only a 46 year supply at a 3% p.a. growth rate. Assuming the more likely pattern of peak output (and consumption) a few decades hence, followed by a gradual decline, the period of substitution of gas for petroleum must be limited to the first few decades of the coming century. After that, alternatives will probably be needed, not only for oil but also for natural gas.

As with oil, there are other potential, but expensive, sources of gas, notably gas dissolved under high pressure in very deep brines and 'frozen' gas (methane clathrates) on the edges of the continental shelves. However, neither of these alternatives is anywhere near practicality, and even the quantities potentially extractable are speculative.

It should be emphasized that prevailing conventional wisdom does not allow for any near-term peaking of either oil or gas. A survey by *The Economist* (April 30-May 6 2005) dismisses the arguments of 'petro-pessimists' (such as Campbell and Laherrere) with brief contempt, arguing that (1) (reported) reserves are still increasing and (2) that this is due to a combination of technology and economics, primarily improvements in oil recovery efficiency. The author of that report ignores all the points made above. Moreover, his optimistic assessment of the potential of new technology for secondary recovery makes no mention of costs.

Implications for growth

It seems indisputable that if petroleum and gas supplies are beginning to experience a long-term supply constraint. In consequence prices are about to rise sharply and although there will be fluctuations, the trend is likely to be upward from now on. The implications for economic growth are consequently even grimmer than I have indicated. The fact that the world seems to have shrugged off the recent rises probably means nothing more than that most users and traders think the price rise since 2004 is temporary, and many still expect a price collapse to follow – as happened in the mid 1980s.

In any case the short-term price elasticity of demand is very small. During the 1970s some financial writers made the mistake of assuming that price elasticity of demand for gasoline was actually zero. They forgot that the cost of fuel is now a small fraction of the cost of owning and operating an automobile or truck. Only those contemplating a long automotive holiday would be likely to modify their plans in the short run. For airlines the cost of fuel is a greater part of the cost of operation, and passing along the increased costs does have an immediate impact on tour operators and peripatetic college students on a tight budget. But those who choose to forego a package tour by air to the Seychelles might actually decide to stay nearer home and use the family car instead.

The more important impact of higher fuel prices is on private decisions to buy new cars (or trucks or aircraft) or tankers, refineries and petrochemical plants. In the late 1970s many US customers decided for the first time to consider fuel economy (mpg) among the criteria for a new car purchase. Congress assisted this process by introducing the corporate

average fuel economy (CAFÉ) standards. Detroit manufacturers half-heartedly and belatedly introduced small cars, such as the Chevette and the Pinto, but they were clunky and poorly made, compared to the Japanese Toyotas and Hondas and the German VWs that were available. A significant share of the US car market was lost to imports in a few years. That had a negative impact on upstream materials and component suppliers, from sheet steel to tires. Declining demand discouraged new investment (and R&D) all along the line and initiated a downward spiral in the US automotive sector that still continues.

The history of the supersonic airliner is another case in point. Prior to the 'energy crisis' of the early 1970s it was assumed by many that the next generation of airliners would be supersonic. The argument was simple: passengers would be willing to pay extra for shorter flights and costly pilots and aircrews could 'produce' many more passenger miles per hour of work, thus boosting labor productivity. The British-French Concorde was merely the first supersonic airliner off the block. The US manufacturers were eagerly planning for faster and bigger SSTs. The sharp increase in petroleum prices after 1973 changed the economics of supersonic flight dramatically. All SST projects were put on hold, then scrapped. Millions of engineering man hours and hundreds of millions of dollars of R&D and engineering investment were wasted.

During the decade 1975-1985 there was a significant shift away from electricity production and towards energy conservation and fuel economy in industrial investment. Prior to 1972 demand for electric power had been increasing at eight percent per annum, and much of that investment was going into nuclear power plants. But higher oil prices triggered higher gas prices – because gas could replace oil in many applications, such as industrial steam generation and home heating. Higher oil and gas prices also triggered higher coal prices, because coal could replace oil or gas in enough applications (notably electric power generation) to affect marginal demand for the fuel. The net result was a shift in aggregate demand away from oil to natural gas (especially in domestic applications) and coal (for electric power generation).

Meanwhile the surge in oil prices after 1973 triggered a sharp economic recession in 1974-75, which cut demand for all forms of energy, including electric power. The slump in demand for electric power, in turn, ended the period of uninterrupted growth and resulted in the termination of a number of (nuclear) power generation projects that were still on the drawing boards, while others were put on hold. Electric power generation investment fell, as retroactive energy conservation investments surged throughout the economy. These ranged from combined heat and power (CHP) installations to industrial heat exchangers and recuperators in chemical plants to better insulation and more efficient lighting in offices and residences. The efficiency of energy (exergy) conversion to useful work increased, for a time, enough to compensate for reduced fuel (exergy) inputs and demand for all fuels (except coal) shifted to a lower growth trajectory.

The period after 1973 also saw a sharp increase in exploration for petroleum and gas, especially outside the OPEC countries. Rapid development took place in Africa, especially Nigeria and Angola. The Gulf of Mexico, the North Slope of Alaska and the North Sea discoveries added to reserves. So, after 1982 the world price of petroleum began to drop and, as it did, the lessons of the 1970s were forgotten. The immediate decline in oil prices (eventually down to \$10 per barrel) trumped the conservation imperative. The CAFÉ standards were not renewed by Congress. Detroit enjoyed a revival, after 1985, based on cheaper oil and growing sales of light (pickup) trucks for family use and, later, the sports utility vehicles (SUVs) that have accounted for most of their profits since 1990. Industrial investment in greater efficiency slowed, in part because the easiest retroactive improvements had already been made.

Once again, the price of oil has risen sharply, since the summer of 2004, and it has happened without any identifiable causal event, such as the Yom Kippur war in 1973, that triggered the Arab embargo. The immediate response has been muted. The conventional wisdom seems to be that it has all happened before, and that higher prices will instantly bring forth new discoveries and new technologies to enhance recovery from old fields and to exploit tar sands and oil shale. The need for radically new technologies to produce useful work without hydrocarbons has not yet penetrated the rosy cloud of reassurances. Certainly, very few investments have been made, so far, even in accelerated exploration, still less in upgrading the petroleum and gas infra-structure and less than that, if possible, in energy conservation or alternatives. In short, the world is currently in denial.

What next? Only time will tell.

Endnotes

1. For instance, RUA (among others) has pointed out in several articles that there is no 'fourth law' (Ayres, Miller 1980; Ayres 1997, 1999). In fact, recycling even low grade wastes is quite possible, in principle, although it might require several successive stages of re-concentration. The only requirement is enough input of available energy (the technical term for available energy is *exergy*). As to the availability question, the solar energy flux impinging on earth is quite sufficient in quantity (albeit low in intensity) to support human civilization indefinitely, even without nuclear power. The low concentration is a technical challenge, but not an ultimate barrier. Moreover, we humans are not limited to what can be collected on the earth's surface. (The moon, and solar satellites, could increase the available area for exergy collection more or less indefinitely).
2. The idea that economic progress is explained mostly by capital investment, while long since abandoned as regards the industrialized countries, was still taken very seriously by many development specialists until very recently. The Harrod-Domar model predicts that the rate of growth of an economy in a year is proportional to the capital investment during the previous year. Harrod intended this as a way of explaining short run fluctuations in output of industrial countries and disavowed its use for developing countries. Yet it was widely adopted by international institutions in the early 1950s for purposes of growth accounting and to estimate the so-called 'financing gap' for developing countries. This capital investment-centered approach was supported by the 'stages of growth' model of W. W. Rostow, who asserted that 'take-off' into sustained growth occurs only when the proportion of investment to national income rises from 5 to 10 percent. (Rostow 1960). Several econometric studies have failed to find any evidence for this theory, however e.g. (Kuznets 1963; United Nations Industrial Development Organization 2003)
3. The unrealistic neglect of materials (and energy) as factors of production in the economic system was pointed out long ago by Boulding (Boulding 1966), Ayres and Kneese (Ayres, Kneese 1969) and Georgescu-Roegen (Georgescu-Roegen 1971). Unfortunately the mainstream view has not adapted. This is extremely significant for policy, in the new century, because if resource consumption is only a consequence – and not a cause – of growth, then 'decoupling' growth from resource consumption is conceptually easy: they were never 'coupled' in the standard theory. On the other hand if increasing resource consumption is inseparable from the 'growth engine' (as we argue), decoupling is impossible and dematerialization will be extremely difficult.
4. Capital stock, in turn, is an accumulation based on investment and depreciation, along the lines of the 'perpetual inventory' approach, which starts from a base year and adds new investments in various categories (e.g. residential housing, non-residential buildings, machinery, roads and bridges, etc.) at current prices adjusted to a standard year, while simultaneously depreciating existing capital stocks based on assumed lifetimes.
5. The major exceptions are multi-sector models built by Dale Jorgenson and his colleagues (Christensen, Cummings, Jorgenson 1983; Gollop, Jorgenson 1980, 1983), using the so-called 'trans-log' production function devised by Lauritz Christenson, Dale Jorgenson and Lawrence Lau (Christensen, Jorgenson, Lau 1973, 1971). Unfortunately these models are extremely data-intensive and lacking in transparency, making them hard to use and interpret.

6. N.B. the national accounts reflect payments only to capital (as interest, dividends, rents and royalties) and to labor (as wages and salaries). The accounts therefore do not explicitly reflect payments to inputs (e.g. energy, raw materials or environmental services from 'nature'). It is possible, of course, to distinguish payments to some tangible resource owners (royalties), and to natural resource extraction labor, but these payments constitute only a very small percentage of the total.
7. Unfortunately when the constraint is eliminated, econometric studies tend to end up with even more unrealistic results (Sylos Labini 1995).
8. Both versions of each variable, r and u have been tested statistically (see (Ayres, Warr 2003).) Both versions are defined and measured in terms of the thermodynamic measure already introduced. The more inclusive definition of resource inputs consistently provides a significantly better fit to the GDP data, regardless of choice of production function. We have done the OLS fits both with and without the constraint of constant returns. Without constant returns, the sum of the three calculated output elasticities turns out to be of the order of 1.3, which is implausibly high.
9. We do *not* assume that firms must operate on, or move along the 'frontier' (by substitution among factors) as they would have to do if they were price-taking profit maximizers operating at the least-cost point with perfect information in a perfectly competitive market. On the contrary, we regard the 'frontier' as the (fuzzy) locus of points in $K-L-R$ space such that firms operating inside at a given time are uncompetitive and likely to decline, whereas firms outside the frontier are more likely to survive and grow. However success or failure in an evolutionary model is not instantaneous, and a firm operating inside the frontier may be able to restructure or innovate to improve its competitive situation. This view is theoretically inconsistent with constant returns, atomistic competition, differentiability, and various other assumptions underlying the notion of the production function (Sylos Labini 1995). For our purposes we rely on the fact that there seems to be an empirical phenomenon that is consistent with the notion of aggregate capital.
10. A somewhat simpler methodology, known as production history forecasting, involves predicting the future shape of the theoretical production curve from the historical data alone. The 'best fit' is then used to predict future production history. This method was systematically applied to all mineral, metal and fuel resources, using U.S. and global data available up to the mid 1970s [Arndt and Roper 1976]. The method does not make use of the additional data (on rates of discovery, proved reserves and so on) used by Hubbert. It is interesting to note that Arndt & Roper's curve-fitting method, using data through 1974, predicted that the peak year would occur in 1984, whereas in fact it had already occurred in 1970. For global production, they predicted a peak in 2034, which now appears very unlikely.

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Figure 1: Feedback cycle

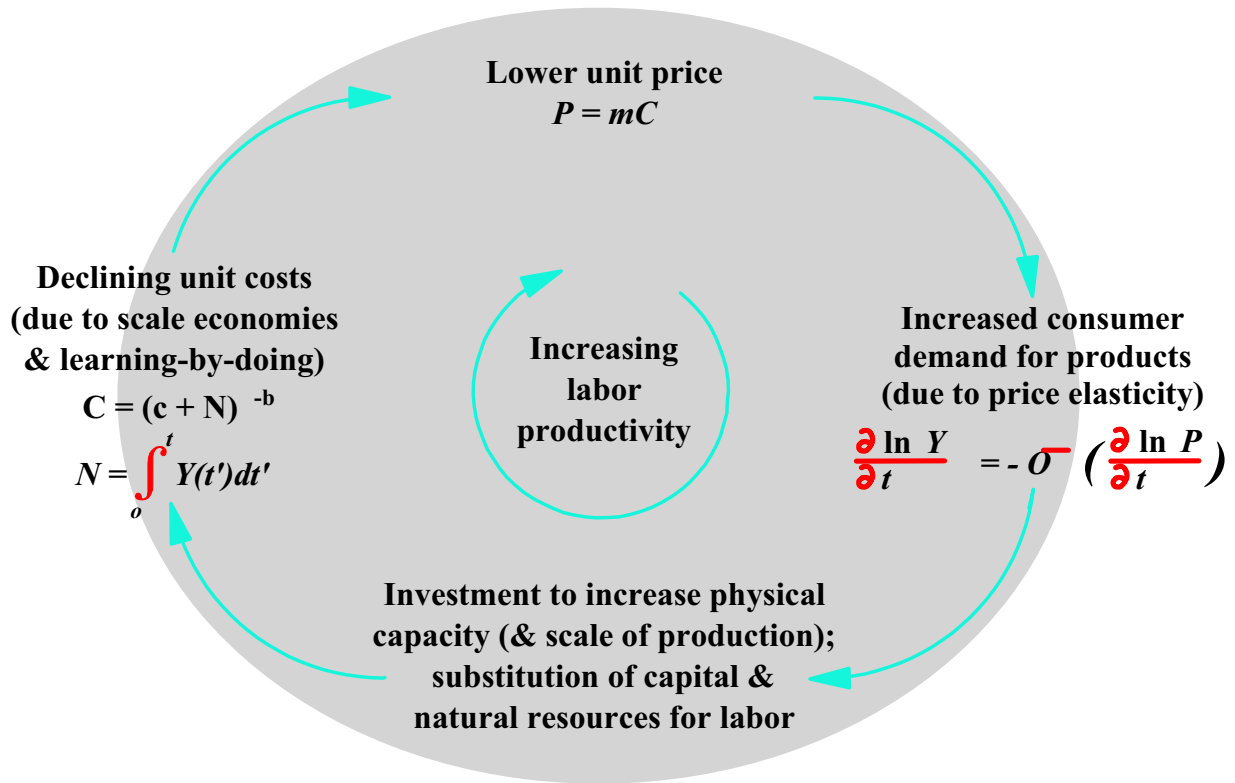
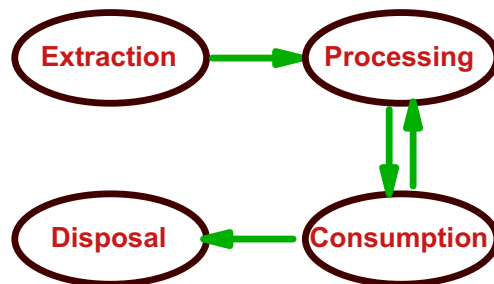


Figure 2: The economic system as a material-energy processor



Value-added by

- utilization of natural "negentropy" stock (Georgescu-Roegen, Daly)
→ system will run down
- utilization of solar energy flux → work
 - embodied exergy
 - embodied information (learning)

Economic growth as a positive feedback process

- resource consumption plays central role

Figure 3: The neoclassical paradigm

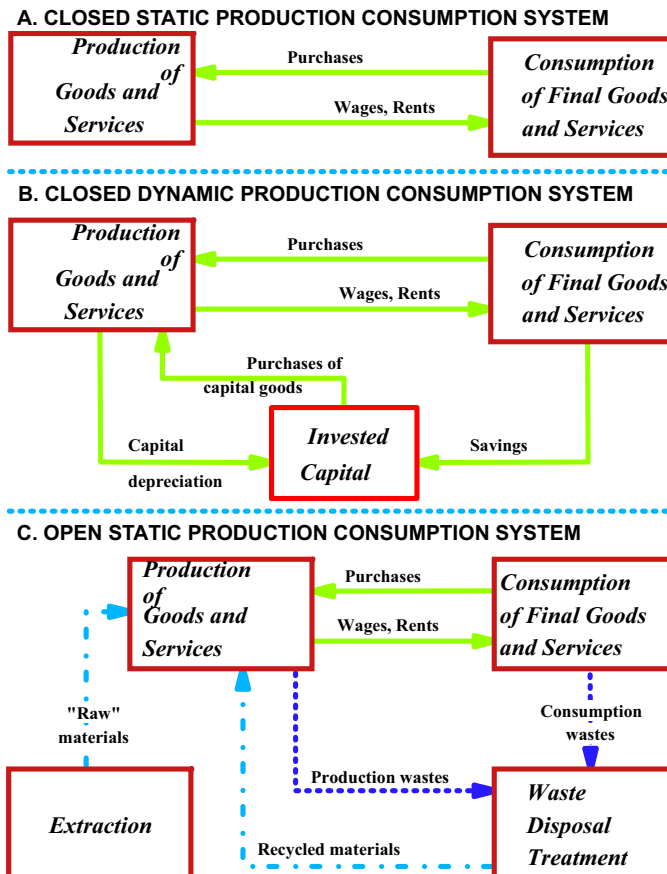


Figure 4: Generalized Cobb-Douglas function

$$Y_t = Q(A_t, H_t K_t, G_t L_t, F_t R_t),$$

$$Y_t = A_t (H_t K_t)^\alpha (G_t L_t)^\beta (F_t R_t)^\gamma$$

Y_t is output at time t , given by Q a function of,

- K_t, L_t, R_t inputs of *capital, labor* and *natural resource services*.
- $\alpha, + \beta + \gamma = 1$, (constant returns to scale assumption)
- A_t is *total factor productivity*
- H_t, G_t and F_t coefficients of *factor quality*

Figure 5: GDP and factors of production USA 1900 - 2000

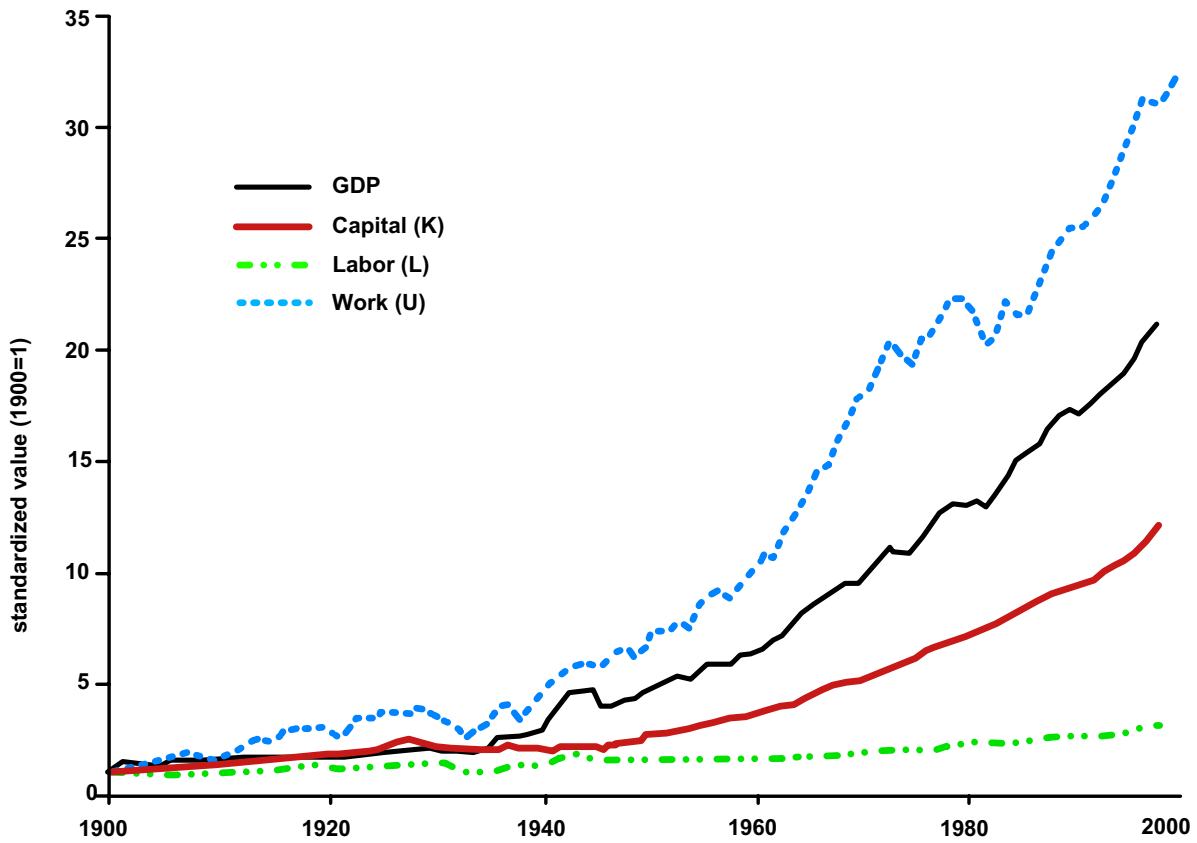


Figure 6: US GDP 1900-1998; Actual vs. 3-factor Cobb Douglas function $L(0.70)$, $K(0.26)$, $E(0.04)$

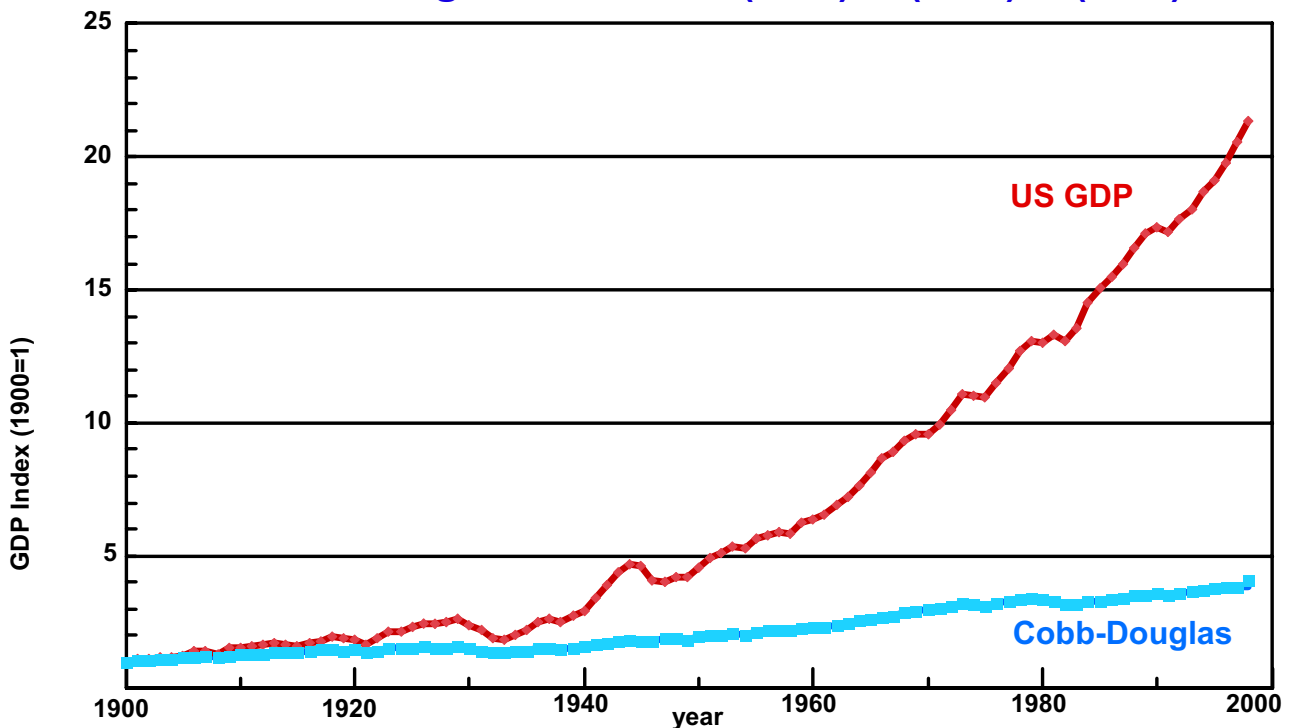


Figure 7: Technological progress function fit

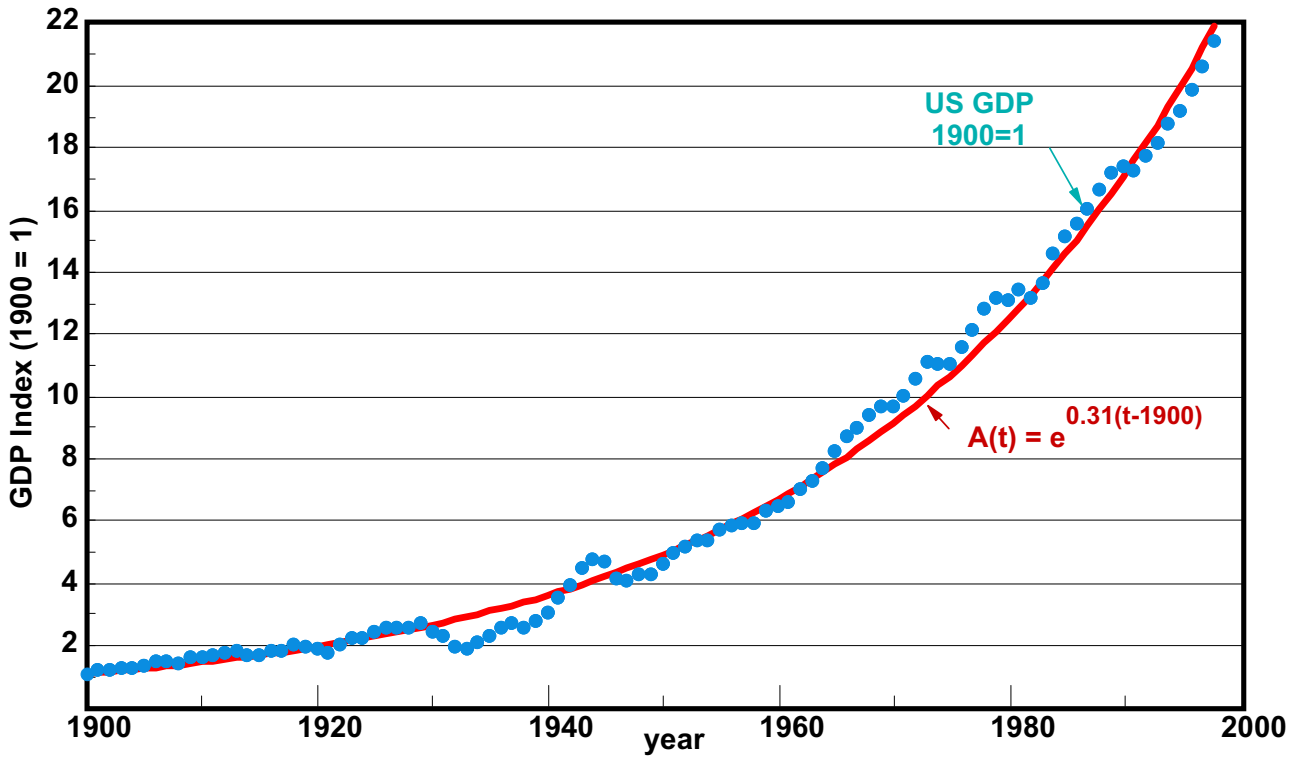


Figure 8: Cost of power per hour as multiple of hourly wage

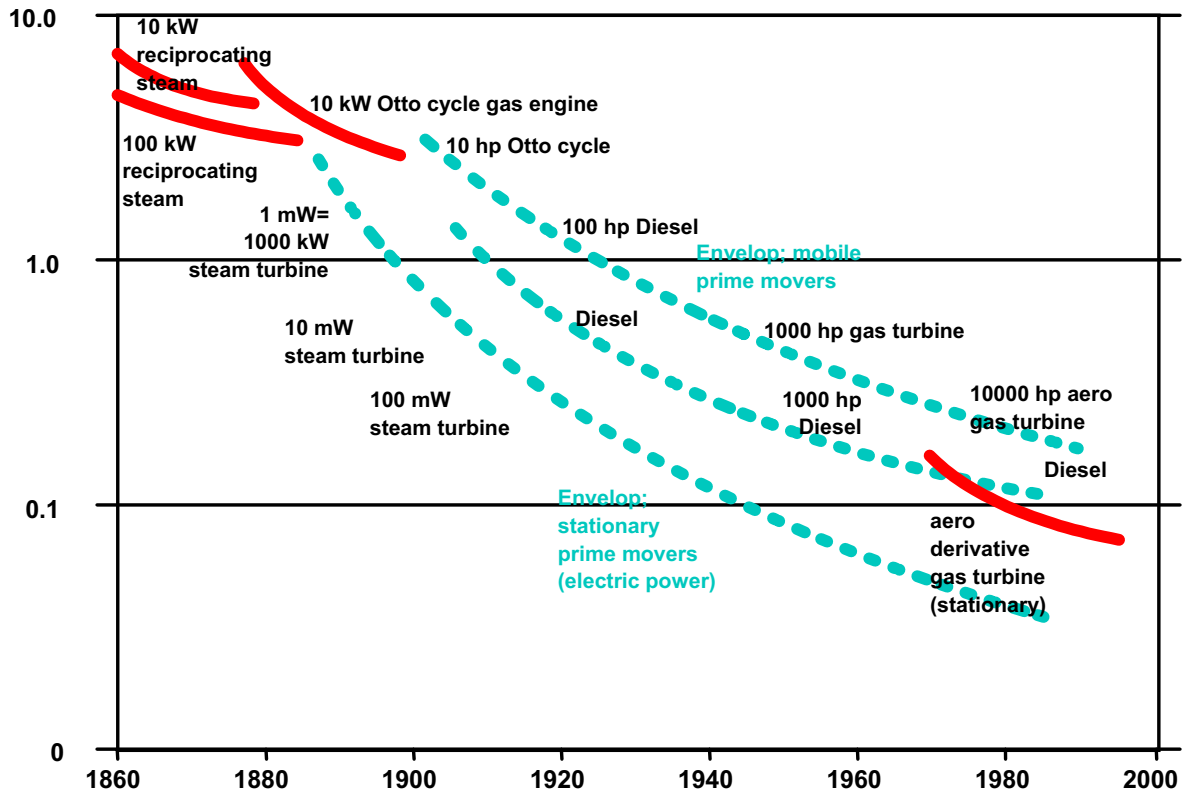


Figure 9: Exergy services (useful work) supply for two definitions of 'useful work', US 1900-2000

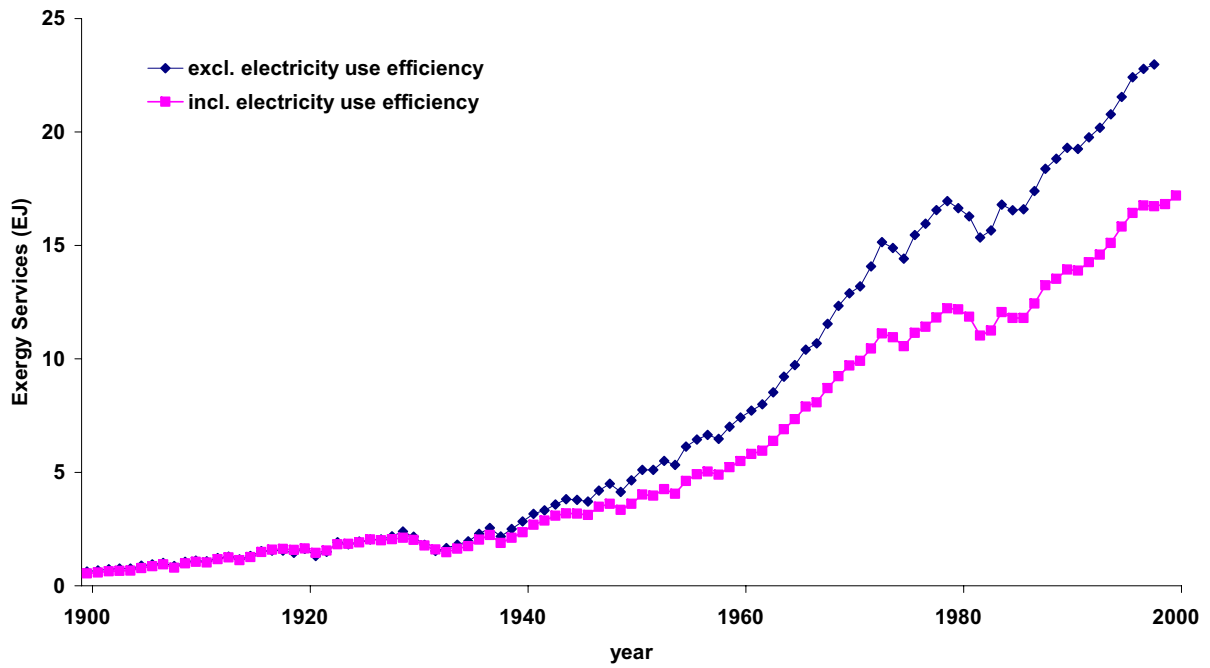


Figure 10: Aggregate Technical Efficiency of Exergy Services for two definitions, US 1900-2000

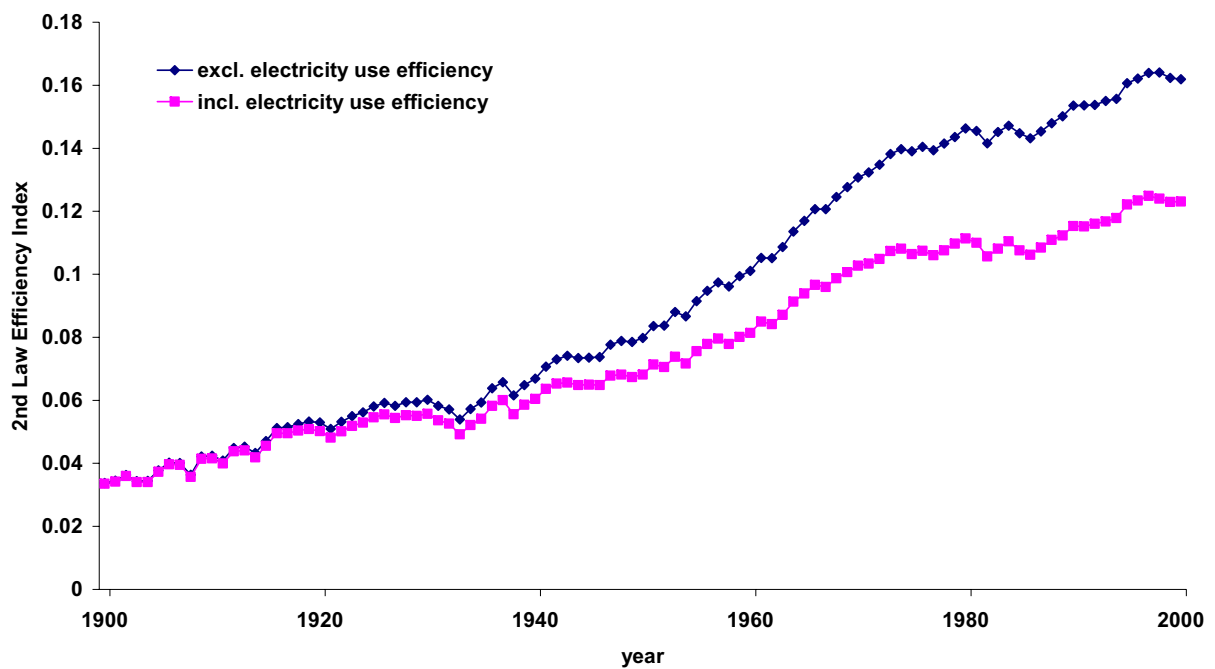
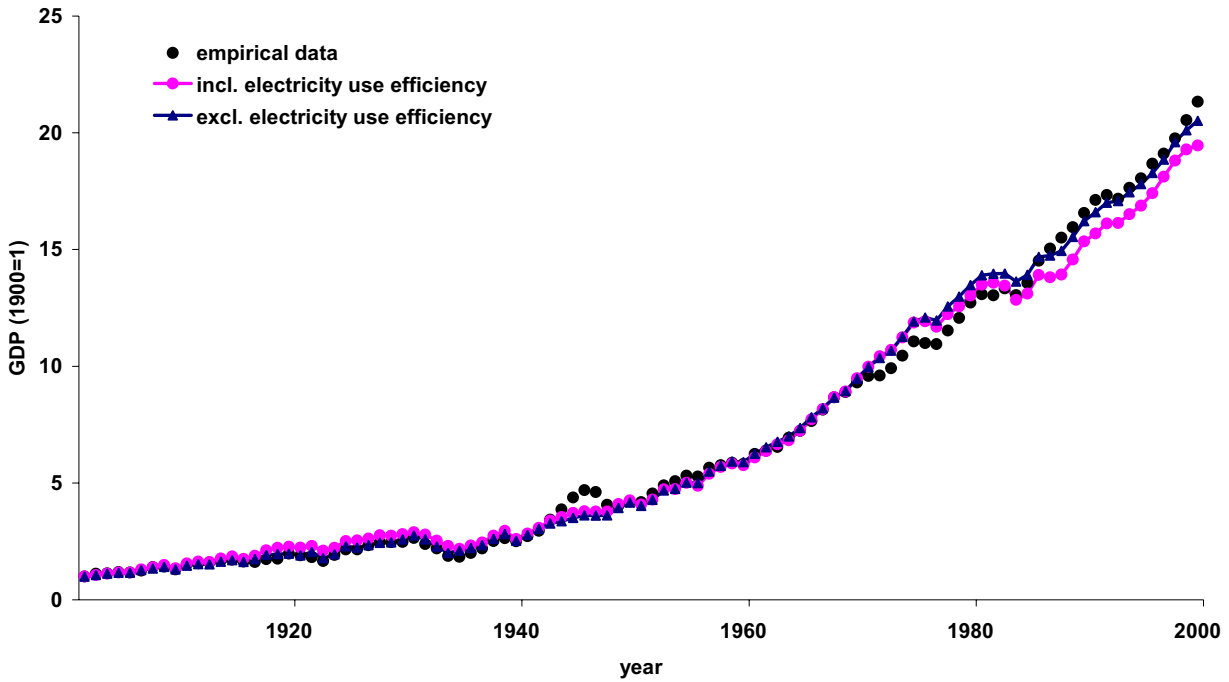
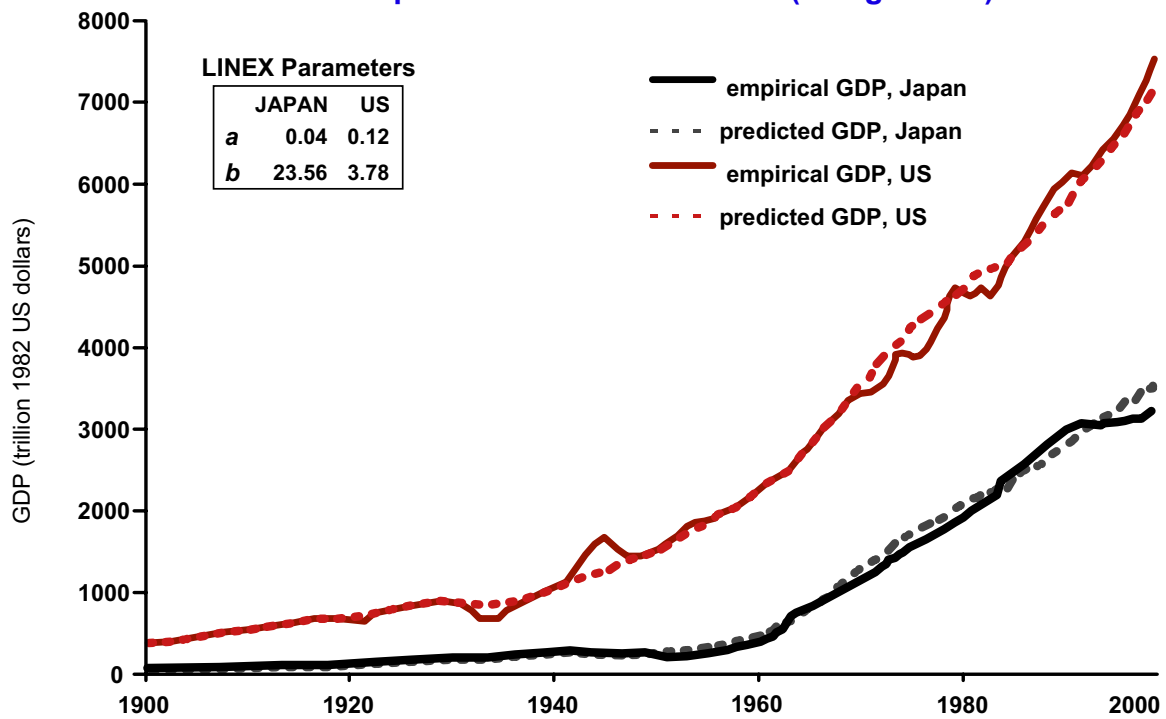


Figure 11: LINEX estimates of GDP for two definitions of Exergy Services

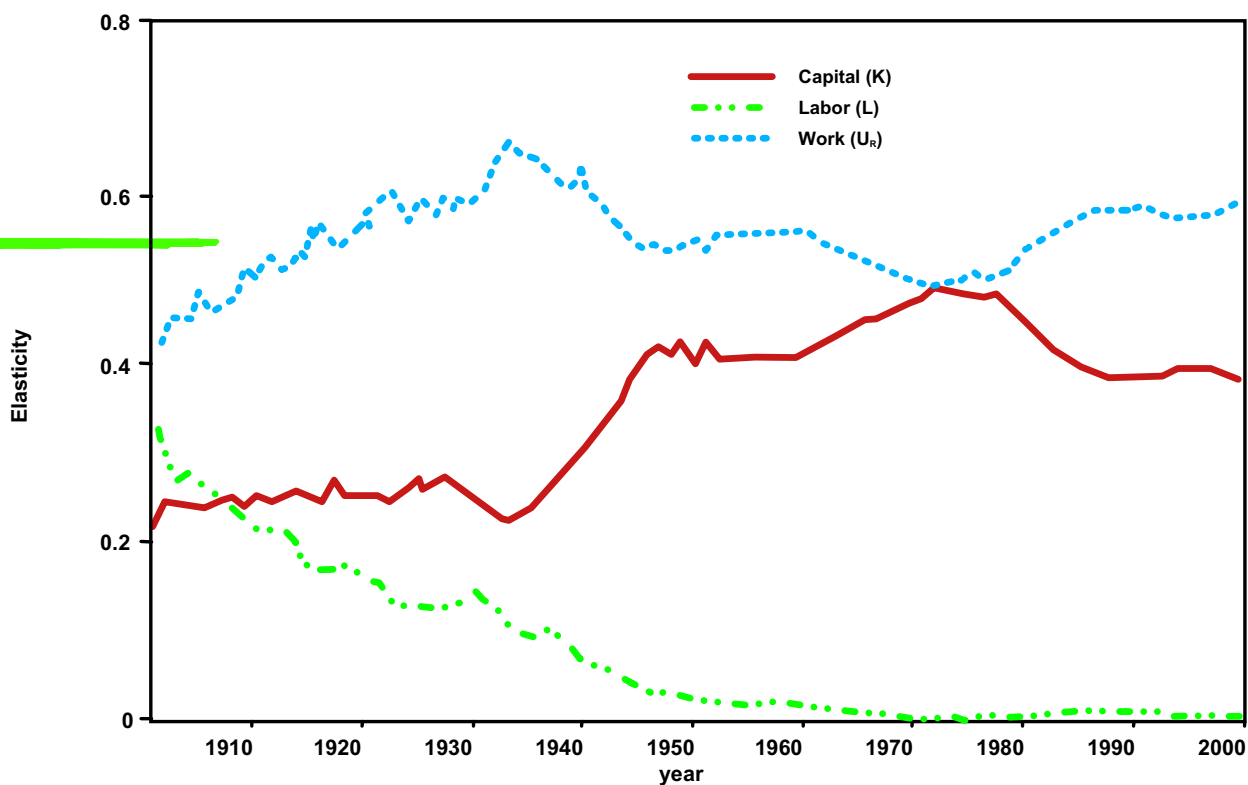


Incl. Electricity LINEX Parameters: $a: 0.093$, $b: 4.527$. Root Mean Square Error: 0.49

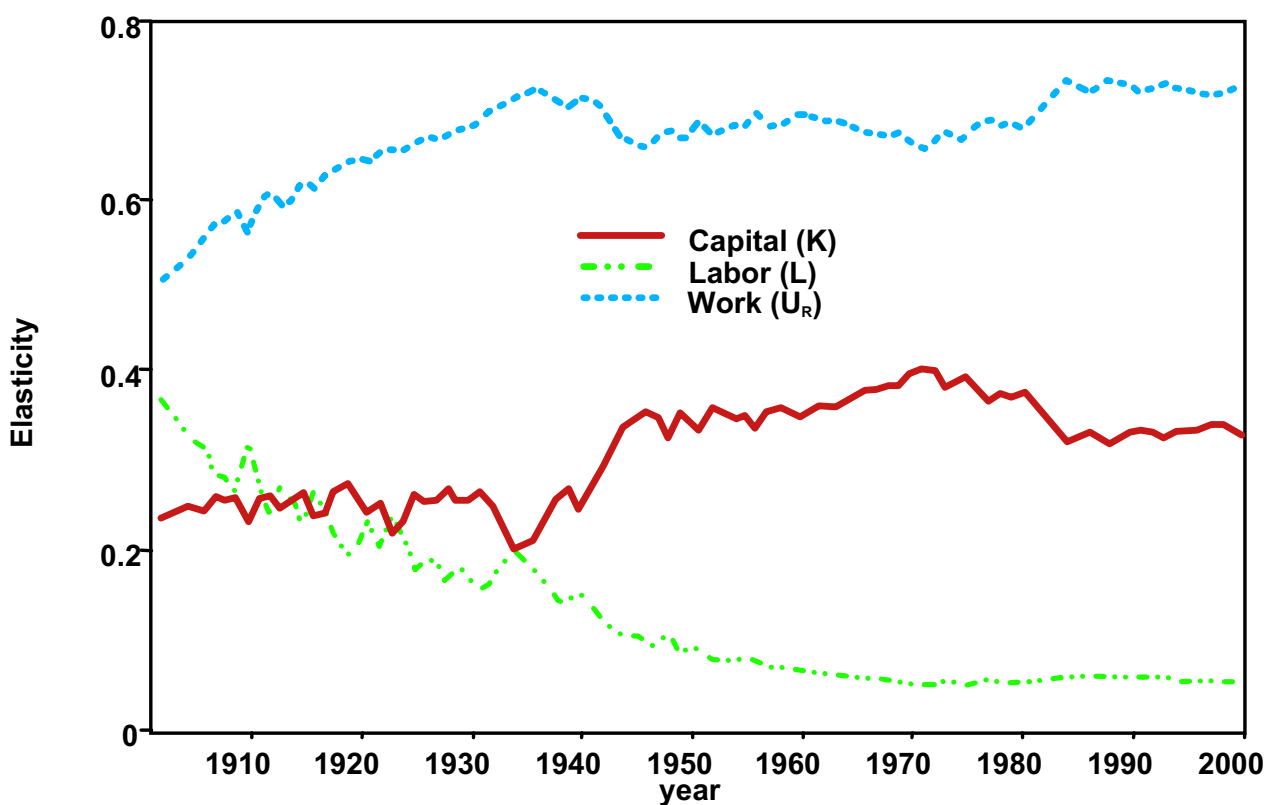
**Figure 12: US and Japan 1900-2000:
Empirical and estimated GDP (using LINEX)**



**Figure 13: LINEX output elasticities, US 1900 - 2000
(for useful excluding electricity use efficiency)**



**Figure 14: LINEX output elasticities, US 1900 - 2000
(for useful work including electricity use efficiency)**



**Figure 15: LINEX output elasticities, Japan 1900 - 2000
(for useful excluding electricity use efficiency)**

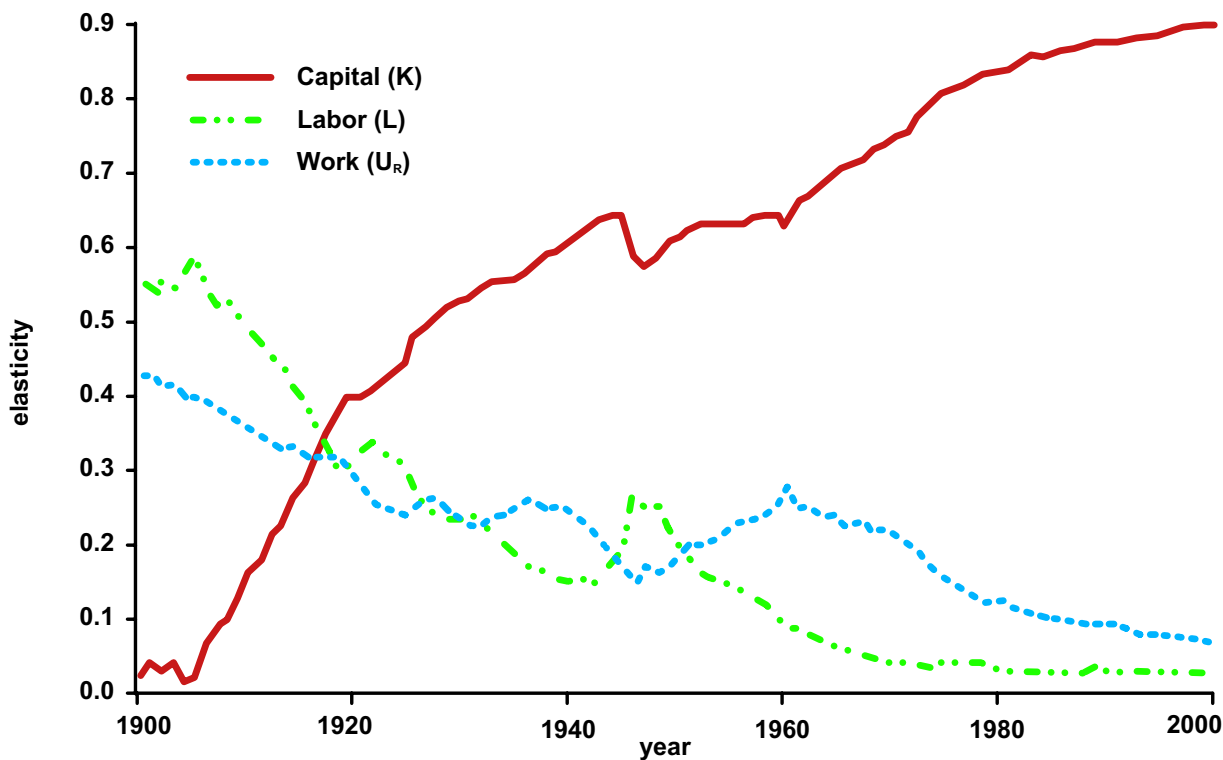


Figure 16: Primary work and primary work/GDP ratio, USA 1900-1998

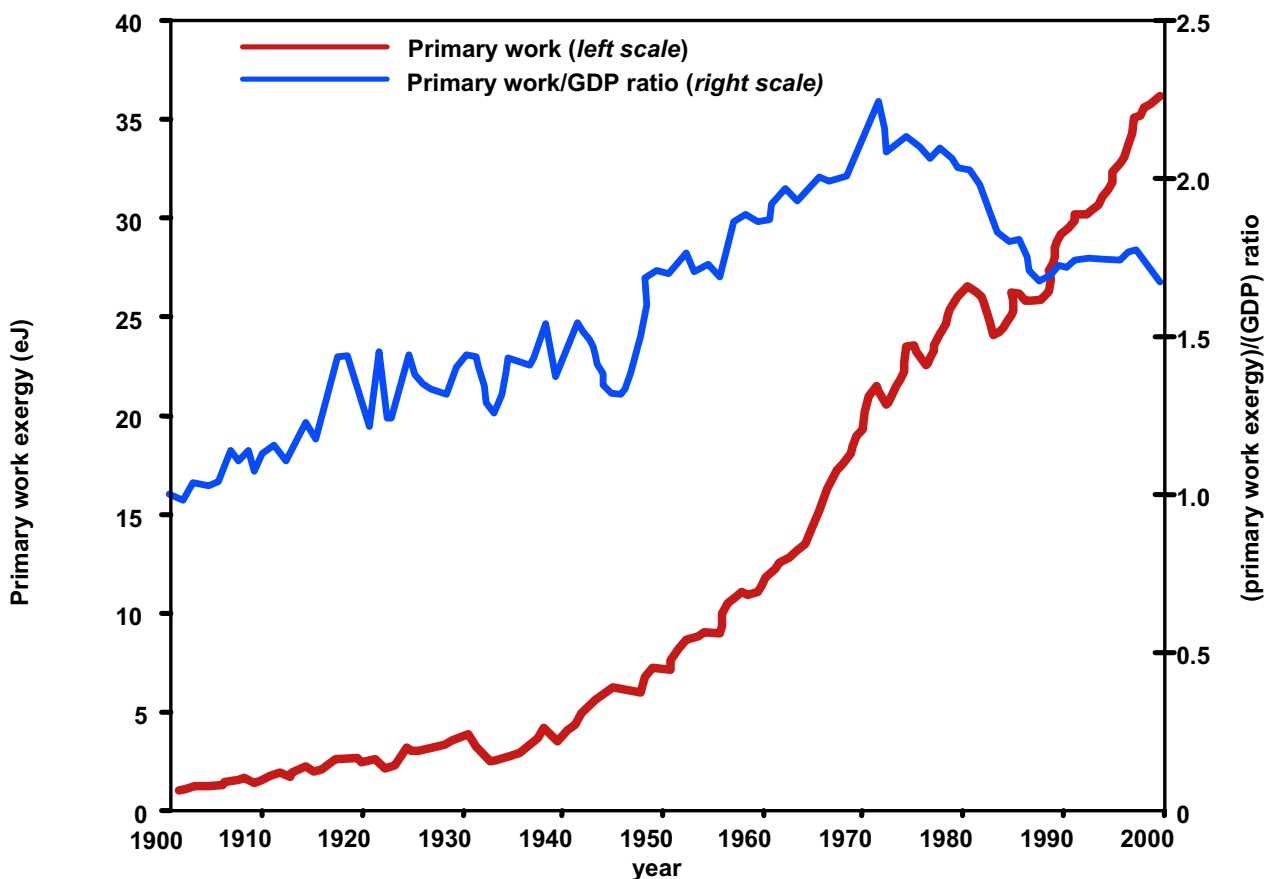


Figure 17: Forecast gross output (GDP): USA 2000-2050

